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TITLE RARE MUON DECAYS AND LEPTON-FAMILY NUMBER CONSERVATION

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RARE MUON DECAYS AND LEPTON-FAMILY NUMBER CONSERVATION

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I. HISTORICAL SURVEY

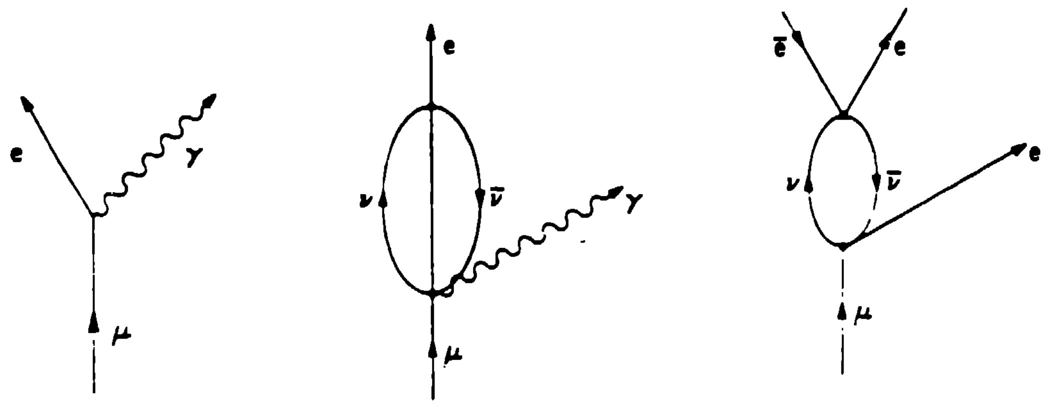
A. Discovery of the Muon

The muon was discovered in cosmic radiation in 1937.[1] For several years it was believed to be the meson of Yukawa's theory that was the carrier of the strong nuclear force: its mass (105.5 MeV/c^2) is deceptively close to Yukawa's predicted meson mass. It was only after it was found that the muon did not interact strongly and another particle (the pi meson) did that the real puzzle presented itself: what role does the muon play? This mystery was succinctly expressed by Rabi: "The Muon, Who Ordered That?"

In fact, this remains one of the central questions in physics to this day, even though the language has changed. Today, one speaks of the family or generation problem, seeking to understand the apparent replication of quark and lepton generations, rather than only why the muon exists.

B. Interest in Neutrinoless Processes

Historically, neutrinoless processes such as $\mu^+ \rightarrow e^+\gamma$, $\mu^+ \rightarrow e^+e^+e^-$, and $\mu^-Z \rightarrow e^-Z$ were of great interest. It was believed that the muon and the electron had identical quantum numbers and so these processes should occur. One can show that a minimal electromagnetic $\mu^+ \rightarrow e^+\gamma$ transition [Fig. 1(a)] violates current conservation.



(a) (b) (c)

Fig. 1. (a) Diagram for a minimal $\mu \to e\gamma$ transition.
(b) Diagram for $\mu \to e\gamma$ mediated by a second-order weak interaction.
(c) Second-order weak-interaction diagram for $\mu \to ee\bar{e}$.

The interaction Lagrangian for Fig. 1(a) is

$$L_{int} = -ej^{\nu}A_{\nu} + \text{Hermitian Conjugate} \tag{1}$$

where $j^{\nu} = \bar{\psi}_e \gamma^{\nu} \psi_{\mu}$.

Current conservation implies

$$\nabla_{\nu} j^{\nu} = 0 = \nabla_{\nu}(\bar{\psi}_e \gamma^{\nu} \psi_{\mu}) = \nabla_{\nu}(\bar{\psi}_e \gamma^{\nu})\psi_{\mu} + \bar{\psi}_e(\nabla_{\nu} \gamma^{\nu} \psi_{\mu}) \quad . \tag{2}$$

The Dirac equation can be written

$$i\nabla_{\nu}\bar{\psi}\gamma^{\nu} + m\bar{\psi} = 0 \tag{3a}$$

and also as

$$i\nabla_{\nu}\gamma^{\nu}\psi - m\psi = 0 \quad , \tag{3b}$$

where m is the mass of the spin-$\frac{1}{2}$ particle involved. Putting (3a) and (3b) into (2), we find

$$0 = \nabla_\nu(\bar{\psi}_e\gamma^\nu)\psi_\mu + \bar{\psi}_e(\nabla_\nu\gamma^\nu\psi_\mu)$$

$$0 = -im_e\bar{\psi}_e\psi_\mu + im_\mu\bar{\psi}_e\psi_\mu = i(m_\mu - m_e)\bar{\psi}_e\psi_\mu \quad , \tag{4}$$

which is inconsistent. Thus, the interaction of Fig. 1(a) violates current conservation and so cannot occur. However, the interaction shown in Fig. 1(b) is allowed. Note that this process is second order in the weak interaction. The calculation of this process yields,[2] for the branching ratio,

$$B_{\mu e\gamma} = \frac{\Gamma(\mu^+ \to e^+\gamma)}{\Gamma(\mu^+ \to e^+\nu\bar{\nu})} = \frac{2\alpha}{3\pi^5}\, G^2 m_p^4 \left(\frac{\Lambda}{m_p}\right)^4 \left(\ell n\frac{2\Lambda}{m_p}\right)^2$$

$$\approx 10^{-15}\left(\frac{\Lambda}{m_p}\right)^4 \left(\ell n\frac{2\Lambda}{m_p}\right)^2 \quad , \tag{5}$$

where Λ is the momentum at which the divergent integral over the neutrino momentum is cut off. It is interesting to note that in the current $\times$ current model, the process $\mu^+ \to e^+e^+e^-$ [Fig. 1(c)] is second-order weak, whereas $\mu^+ \to e^+\gamma$ [Fig. 1(b)] is further suppressed by order α. The small rate implied by (5) did not confront the experimental upper limit, $B_{\mu e\gamma} < 2 \times 10^{-5}$, in 1957.[3]

A fundamental problem with the current $\times$ current model of the weak interaction is that the interaction grows with energy and ultimately violates unitarity. A solution to this problem was the introduction of the intermediate vector boson (now called $W^\pm$) proposed by Schwinger in 1957.[4] In this theory, the decay $\mu^+ \to e^+\gamma$ can proceed via the diagram shown in Fig. 2. The rate for this

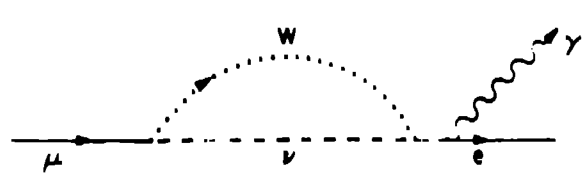


Fig. 2. Diagram for $\mu \to e\gamma$ in intermediate vector boson model.

diagram is second order in the semiweak coupling constant of the W to the leptons and so is first-order weak. $B_{\mu e\gamma}$ is given by[5]

$$B_{\mu e\gamma} = \frac{3}{8\pi} \alpha N^2(m_W, \Lambda) \tag{6}$$

$N^2 = 1$ for $\Lambda = m_W$ provided that the W has no anomalous magnetic moment. In general, N varies rapidly with Λ/m_W.

The large $B_{\mu e\gamma}$ implied by (6) was in conflict with the measured upper limit. In order to explain this, Ebel and Ernst[5] noted that for an anomalous moment near 0.75, $B_{\mu e\gamma}$ is nearly zero for $0.1 \lesssim \Lambda/m_W \lesssim 100$. However, the calculated rates for $\mu^+ \to e^+e^+e^-$ and $\mu^-Z \to e^-Z$ are not similarly suppressed. Thus, a dilemma existed. We could conclude that neutrinoless μ-e transitions were either suppressed by some dynamical mechanism or forbidden by some conservation law. Incidentally, this historical example of the need to measure all possible neutrinoless μ-e transitions will be repeated.

C. Lepton-Number-Conservation Laws

The first lepton-number conservation law was proposed by Konopinski and Mahmoud in 1953.[6] In this scheme, e^- and μ^+ are each assigned $L = +1$, whereas e^+ and μ^- have $L = -1$. The assignments are summarized in Table I. There is a single conserved quantity, ΣL. This is an extremely economical scheme that forbids $\mu^+ \to e^+\gamma$, $\mu^+ \to e^+e^+e^-$, $\mu^-Z \to e^-Z$, $K^0 \to \mu^\pm e^\mp$ but allows $\mu^-Z \to e^+(Z-2)$. An early problem with this arrangement was that it predicted that identical neutrinos emerge from muon decay ($\mu^+ \to e^+\nu\nu$), giving a value of zero to the Michel parameter ρ, in contradiction to the measured $\rho \approx 3/4$. The scheme can be patched up by allowing distinct ν_e and ν_μ, but it cannot be extended to a third lepton generation.

The additive lepton number scheme was introduced in 1957 by Schwinger, by Nishijima, and by Bludman.[7] The assignments are shown in Table II. The conserved quantities are ΣL_μ and ΣL_e. This forbids all neutrinoless μ-e transitions, as well as $K_L \to \mu^\pm e^\mp$ and $K^+ \to \pi^+\mu^\pm e^\mp$. The extension of this scheme to include the τ lepton generation is straightforward.

The multiplicative scheme was put forth by Feinberg and Weinberg[8] in 1961. The assignments are given in Table III. The conserved quantities here are ΣL and ΠL_P. This choice forbids all neutrinoless processes but allows $\mu^+ \to e^+\bar{\nu}_e\nu_\mu$, as well as $\mu^+ \to e^+\nu_e\bar{\nu}_\mu$.

These schemes require distinct muon- and electron-type neutrinos. In 1959-60, Pontecorvo and, independently, Schwartz[9] proposed

TABLE I. Konopinski-Mahmoud Lepton-Number Assignments

L		
+1	e^-, μ^+, ν	
-1	e^+, μ^-, $\bar{\nu}$	Conservation Law: ΣL = Constant
0	Everything else	

Forbids:		Allows:	
	$\mu^+ \to e^+\gamma$		$\mu^-Z \to e^+(Z-2)$
	$\mu^+ \to e^+e^+e^-$		$(\mu^+e^-) \to (\mu^-e^+)$
	$\mu^-Z \to e^-Z$		$K^+ \to \mu^+e^+\pi^-$
	$K^0 \to \mu^\pm e^\mp$		$\mu^+ \to e^+\nu\nu$

TABLE II. Additive Lepton-Number Assignments

L_e		L_μ		L_τ	
+1	e^-, ν_e	+1	μ^-, ν_μ	+1	τ^-, ν_τ
-1	e^+, $\bar{\nu}_e$	-1	μ^+, $\bar{\nu}_\mu$	-1	τ^+, $\bar{\nu}_\tau$
0	Everything else	0	Everything else	0	Everything else

Conservations Laws: ΣL_e = Constant

ΣL_μ = Constant

ΣL_τ = Constant

Forbids:		Allows:	
	$\mu^+ \to e^+\gamma$		$\mu^+ \to e^+\nu_e\bar{\nu}_\mu$
	$\mu^+ \to e^+e^+e^-$		
	$\mu^-Z \to e^-Z$		
	$\mu^-Z \to e^+(Z-2)$		
	$K^0 \to \mu e$		
	$\mu^+ \to e^+\bar{\nu}_e\nu_\mu$		

accelerator experiments to determine if these neutrinos are indeed distinct. Antineutrinos produced in reactors result from β decay and so are electron-antineutrinos. When they interact with matter they produce positrons. Neutrinos produced at accelerators come predominantly from $\pi^+ \to \mu^+\nu_\mu$ and so should produce muons when they

TABLE III. Multiplicative Lepton-Number Scheme

L		L_p	
+1	μ^-, e^-, ν_e, ν_μ	+1	e^+, $\overset{(-)}{\nu}_e$
-1	μ^+, e^+, $\bar{\nu}_e$, $\bar{\nu}_\mu$	-1	μ^+, $\overset{(-)}{\nu}_\mu$
0	Everything else	+1	Everything else

Conservation Laws: ΣL = Constant

ΠL_p = Constant

Forbids:	Allows:
$\mu^+ \to e^+\gamma$	$\mu^+ \to e^+\nu_e\bar{\nu}_\mu$
$\mu^+ \to e^+e^+e^-$	$\mu^+ \to e^+\bar{\nu}_e\nu_\mu$
$\mu^-Z \to e^-Z$	$(\mu^+e^-) \to (\mu^-e^+)$
$\mu^-Z \to e^+(Z-2)$	
$K^0 \to \mu e$	

interact. This was indeed found to be the case in the famous "two-neutrino experiment" in 1962.[10] This stunning result validated the concept of lepton-number conservation and virtually closed the book on further searches for neutrinoless μ-e transitions for some time.

The situation was summarized in a 1963 paper by S. Frankel et al.[11]: "The results of the neutrino experiments...indicate that the normal weak interaction channels are closed to this decay mode ($\mu \to e\gamma$). Since it now appears that this decay is not lurking just beyond present experimental resolution, any further search...seems futile." The experimental status as of 1964 and 1975 is given in Table IV.

II. PRESENT VIEW OF LEPTON-NUMBER CONSERVATION

A. Nature of Conservation Laws

There are two kinds of conservation laws: those which are related to space-time translations or rotations and those which are not. Examples of the first kind are conservation of energy, momentum, and angular momentum. The second type includes conservation of electric charge, baryon number, and lepton number. We believe the first type of conservation law is fundamental. In 1933 Pauli[12] postulated the existence of what appeared to be an unobservable

TABLE IV. Experimental Status of Various Lepton-Family-Violating Processes in 1964 and 1975

Process	Upper Limit in 1964 (90% CL)	Upper Limit in 1975 (90% CL)
$BR(\mu^+ \to e^+\gamma)$	2.2×10^{-8}	2.2×10^{-8}
$BR(\mu^+ \to e^+e^+e^-)$	1.2×10^{-7}	6.2×10^{-9}
$BR(\mu^+ \to e^+\gamma\gamma)$	1.6×10^{-5}	4×10^{-6}
$\dfrac{\Gamma(\mu^-Z \to e^-Z)}{\Gamma(\mu^-Z \to \nu[Z-1])}$	2.4×10^{-7} (Cu)	2×10^{-8} (Cu)

particle (the neutrino), rather than give up conservation of energy and angular momentum in beta decay.

In general, the second kind of conservation law is regarded as less fundamental. However, the conservation of electric charge is related to gauge invariance of the electromagnetic field and its associated massless gauge boson (the photon). No such massless gauge boson exists for baryon or lepton number, and so it has been argued that exact conservation of these quantities is absurd. A heuristic argument presented by de Rujula, Georgi, and Glashow[13] demonstrates this.

Consider a region of space from which you are excluded. If someone throws an electrically charged object into the region, can you detect this event by observations made outside the region? Yes, you can. The memory of the electric charge is preserved by the electric field outside the region: the fact that the photon is massless implies that the field extends over all space. Now, if someone throws a lepton (or a baryon) into the region, can you tell? No! The lepton leaves no trace at all. Thus, we do not expect lepton number to be an exact global symmetry.

How do we know that there is no massless gauge boson coupled to lepton number? Of course, we have not seen one but that does not constitute proof. The best theoretical argument was given by Lee and Yang in 1955.[14] They showed that such a massless gauge boson would violate the gravitational equivalence principle (Eötvös experiment). Consider a massless vector field coupled to lepton number with coupling constant η. The force between two massive objects then includes a contribution from the Coulomb-like force between the leptons:

$$F_G = -\frac{GM_1M_2}{R^2} + \frac{\eta^2 Z_1 Z_2}{R^2} = -\frac{G}{R^2} M_{1G}\, M_{2G} \quad . \tag{7}$$

The gravitational mass, M_{1G}, is supposed to be equivalent to the inertial mass, M_1. But M/Z varies greatly from substance to substance. Thus, the equivalence principle cannot hold for all substances. The experimental precision on the equivalence principle corresponds to an upper limit on $\eta/\sqrt{G}$.

Thus, we do not expect lepton or baryon numbers to be exactly conserved. Nevertheless, the experimental evidence shows that any violation of conservation of these quantities is extremely weak. In fact, it is a theoretical problem to explain why the violation is so weak if it is not zero.

B. The "Standard Model"

The standard model is a spontaneously broken gauge theory of electroweak and strong interactions with a group structure $SU(3)_c \times SU(2)_L \times U(1)$.[15] The fermions are placed in left-handed doublets and right-handed singlets. In this model, the neutrinos are massless, and there is no lepton number (either total lepton number or separate electron, muon and tau number) violation. The standard model is a minimal model in the sense that it includes only those elements which are required by present data. Many elements (such as exact V-A weak interactions) are put in explicitly, and there is no explanation for the replication of the family structure, nor is there any way to calculate the many parameters (coupling constants, masses, mixing angles) of the theory. Nevertheless, this model is in agreement with all observations.

C. Heavy Neutrinos

The simplest extension of the minimal Weinberg-Salam-Glashow model would be the inclusion of neutrino masses. In such a model, the neutrinos could mix and give rise to (for example) $\mu \to e\gamma$.[16] The branching ratio, $B_{\mu e\gamma}$, is given by

$$B_{\mu e\gamma} = \frac{3\alpha}{32\pi} \left|U^*_{23}U_{13}\right|^2 \frac{(m_3 - m_2)^2(m_3 - m_1)^2}{m_W^4} \quad , \tag{8}$$

where m_i is the mass of the neutrino in the i^{th} generation and U_{ij} is an element of the unitary neutrino mass mixing matrix. Using the present upper limit for the tau-neutrino mass ($m_{\nu_\tau} < 250$ MeV) and $|U^*_{23}U_{13}|^2 < 3 \times 10^{-3}$, we find $B_{\mu e\gamma} < 1 \times 10^{-16}$. For a further lepton generation with maximal mixing, a neutrino mass of 1.8 GeV would result in $B_{\mu e\gamma} = 10^{-10}$. Note that this model has succeeded in suppressing muon number violating processes without needing to impose a conservation law.

In this model, $\mu^+ \to e^+e^+e^-$ is dominated by Z^0 exchange. Here,

$$\Gamma(\mu \to ee\bar{e})/\Gamma(\mu \to e\gamma) = \frac{2\alpha}{\pi} \ln^2 \left[\frac{(m_3 - m_2)(m_3 - m_1)}{m_W^2}\right] . \qquad (9)$$

Models with doubly charged leptons can also have muon-number violation but with $B_{\mu 3e}$ larger than $B_{\mu e\gamma}$.

D. Expanded Higgs Sector

Another straightforward extension of the standard model is the inclusion of more than one doublet of Higgs bosons.[17] The standard model requires at least one Higgs doublet to generate masses for the vector bosons. However, there is considerable freedom in the nature of the Higgs sector. Muon-number violations can be mediated by the multiple Higgs doublets.

The Higgs can contribute through one-loop diagrams and two-loop diagrams. The two-loop is larger than the one-loop contribution if the Higgs mass, m_H, is greater than 3 GeV. If $m_H \ll m_W$, then

$$B_{\mu e\gamma} \simeq 3\left(\frac{\alpha}{\pi}\right)^3 \simeq 4 \times 10^{-8} \quad , \qquad (10)$$

which exceeds present experimental bounds. If $m_H > m_W$,

$$B_{\mu e\gamma} \sim \left(\frac{\alpha}{\pi}\right)^3 \left(\frac{m_W}{m_H}\right)^4 \quad . \qquad (11)$$

For $m_H < 1$ TeV (this is current theoretical prejudice), $B_{\mu e\gamma} > 4 \times 10^{-13}$. Note that by studying very rare processes, one is exploring extremely large mass scales. Finally, the rate for $\mu \to ee\bar{e}$ is smaller than the rate for $\mu \to e\gamma$ but by less than a factor of α/π as would be expected from Dalitz pair production.

E. Other Theoretical Models

Space does not permit a discussion of the many other extensions to the standard model which result in lepton-family-number nonconservation. Examples include existence of flavor-changing neutral gauge bosons (for example, the gauge bosons associated with horizontal gauge interactions,[18] or the gauge bosons present in extended technicolor theories[19]); composite models[20]; muon-number violation mediated by light lepto-quarks (present in some grand unified theories[21] and in extended technicolor theories[19]); muon-number violation mediated by supersymmetric partners of the usual $SU(2)_L \times U(1)$ gauge bosons[22]; existence of new electroweak interactions.[23] In general, these different sources of lepton-number nonconservation predict different relative strengths for the various neutrinoless transitions. This result underscores the importance of searching for all of these processes.

Another process which violates lepton-family-number conservation is neutrino oscillations.[24] Oscillations explicitly require massive neutrinos, whereas this is not the case for the processes discussed above. However, oscillation experiments can be sensitive to very small neutrino masses (<1 eV), whereas effects in the neutrinoless transitions caused by those masses alone would be negligibly small.

III. THE SEARCH FOR $\mu \to e\gamma$

A. Rare Decays

In order to search for any rare decay, one needs a copious source of the decaying particle and an apparatus that can detect the decay products. Furthermore, the experiment must be capable of eliminating possible background processes and of identifying the desired reaction.

In the absence of background or signal, the 90% confidence level upper limit is given by

$$B(90\%\ CL) = \frac{2.3}{N_D \times \Omega/4\pi \times \varepsilon} , \qquad (12)$$

where N_D = number of parent particles decaying, $\Omega/4\pi$ = fractional solid-angle acceptance of the detector, and ε = overall detector efficiency.

N_D increases linearly with running time, so the limit improves linearly with running time until some background is encountered. In the presence of background, the limit improves with the square root of the running time because one must subtract the number of background events from the number of observed events, and the statistical uncertainties in these numbers determine the limit.

B. History

The first searches for the $\mu \to e\gamma$ decay were performed by Hincks and Pontecorvo, and Sard and Althaus in 1948[25] before the true nature of muon decay was understood. The energy of the particles was measured using Geiger-Mueller tubes and absorbers. An upper limit of $B_{\mu e\gamma} < 5 \times 10^{-2}$ was obtained. Subsequent measurements[3,26] used a variety of techniques (range measurements with scintillation counters, water Čerenkov counters, spark chambers, a Freon bubble chamber, and energy measurements with NaI crystals). The upper limit, as a function of time, is shown in Fig. 3.

Due to the long lifetime of the muon (2.2 μs), all studies of muon decay have been done with stopped muons. The decay $\mu^+ \to e^+\gamma$ (μ^+ are studied because stopped μ^- can be captured in the stopping target) is characterized by a monochromatic positron and photon (Energy = 52.83 MeV), which are emitted simultaneously at 180° with respect to each other. It is these distinguishing attributes which must be used to search for $\mu \to e\gamma$.

Early measurements used range to determine the energy of the positron and the converted photon. Later experiments used NaI crystals or a magnetic spectrometer. The precision of the energy measurements was poor. As a result, these experiments were limited by the background from the decay $\mu^+ \to e^+\nu_e\bar{\nu}_\mu\gamma$ where the two neutrinos have little energy and at e^+-γ are nearly acollinear. The branching ratio in this configuration[27] is

$$B_{rad} = \frac{\Gamma(\mu^+ \to e^+\nu\bar{\nu}\gamma, \theta_{e\gamma} \approx 180^\circ)}{\Gamma(\mu^+ \to e^+\nu\bar{\nu})} =$$

$$\frac{\alpha}{2\pi}\left[(1 - x)^2 + 4(1 - x)(1 - y)\right]y\, dy\, dx\, d(\cos\theta_{e\gamma}) \quad , \qquad (13)$$

where $\theta_{e\gamma}$ is the angle between the positron and the photon and x

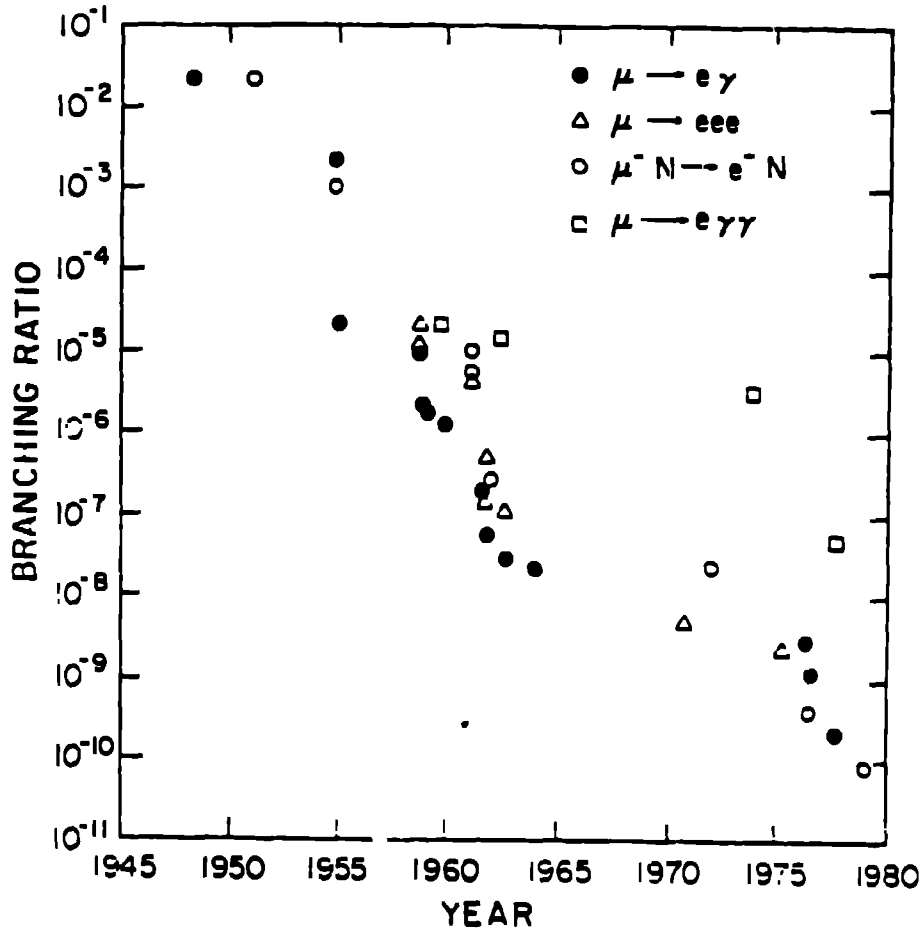


Fig. 3. Upper limit for $B_{\mu e\gamma}$ and several other muon-number violating processes as a function of time.

and y are the positron and photon energies, respectively, in units of $m_\mu/2$. Thus, for $\Delta\theta_{e\gamma} = 0.1$, $\Delta x = 0.1$, and $\Delta y = 0.5$ (typical of experiments performed around 1963), $B_{rad} = 1.4 \times 10^{-8}$. This is about as well as these experiments did. However, the modest μ^+ intensities (derived from stopped π^+'s) did not permit more sensitive searches to be made. The advent of high intensity pion beams and muon beams at meson factories, coupled with substantial detector improvements, allowed more sensitive experiments to be performed.

C. The SIN Experiment of 1977

This experiment[28] is largely responsible for the renaissance in interest in processes violating muon number conservation. An ill-founded rumor of a $\mu \rightarrow e\gamma$ signal, which proved to be incorrect, was the cause. This experiment was an order of magnitude more sensitive than any previous experiment. The apparatus is shown in Fig. 4. The major detecting device is two large NaI crystals located on opposite sides of the stopping target. This design is essentially identical to that used by Frankel et al.[26] in 1963. This arrangement had a relatively small acceptance ($\Omega/4\pi = 1.2\%$ for $\mu \to e\gamma$ events). The experiment utilized a 90 MeV/c μ^+ beam with a moderator before the scintillation target. The stopping rate was $\sim 5 \times 10^5$ μ/s. The NaI energy resolution was (4.6 ± 0.4) MeV (FWHM). Good shower containment was achieved by using large NaI detectors (27.7 cm diameter, 33 cm long) and collimating the entrance aperture. The improved energy resolution implies that the internal bremsstrahlung background is comfortably below 10^{-9}. An upper limit $B_{\mu e\gamma} < 1.0 \times 10^{-9}$ (90% CL) was achieved.

A nearly identical experiment was performed at the same time at TRIUMF.[29] The beam intensity was 2×10^5 π^+/s. An upper limit of $B_{\mu e\gamma} < 3.6 \times 10^{-9}$ (90% CL) was obtained.

These two experiments improved the upper limit for $B_{\mu e\gamma}$ by an order of magnitude using an experimental design that is essentially identical to previous experiments. The improvement was possible because of improvements in the incident beam, in the NaI detectors, and in the electronics. Nevertheless, this limit is about as low as one can set with this technique. Further improvement requires different techniques.

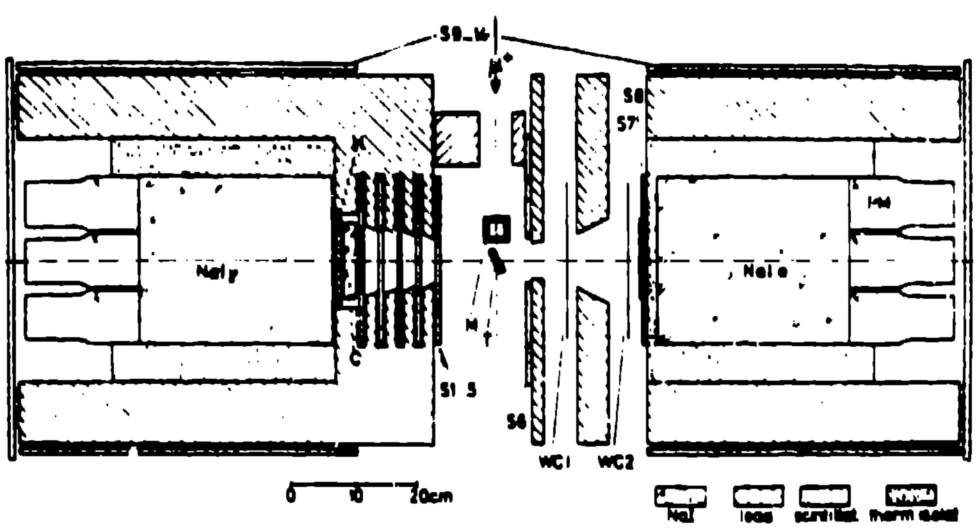


Fig. 4. Schematic diagram of the SIN $\mu \to e\gamma$ experimental apparatus.

D. The LAMPF Experiment of 1978-79

A plan view of the apparatus for this experiment[30] is shown in Fig. 5. There were several new techniques used in this experiment, which will be described below. These are a magnetic spectrometer to measure the e^+ vector momentum, a segmented NaI detector to measure the photon energy and conversion point, and a muon beam of very low energy ("surface" beam).

The goal of this experiment was to be sensitive to $B_{\mu e\gamma}$ at the 10^{-10} level. Since $\Omega/4\pi$ for this detector is ~2%, more than 10^{12} muon decays were required. Furthermore, a thin stopping target was required to minimize multiple scattering, bremsstrahlung, and annihilation of the outgoing positron.

The experiment used a surface muon beam. The muons in such a beam originate from pions that come to rest near the surface of the pion production target. These muons come from a small, well-defined source. They may be imaged by simple beam optics. The muon acceptance is high, the momentum is low (28 MeV/c), the momentum dispersion ($\Delta p/p$) and spot size are both small, and the intensity is high. The range of the surface muons in polyethylene is

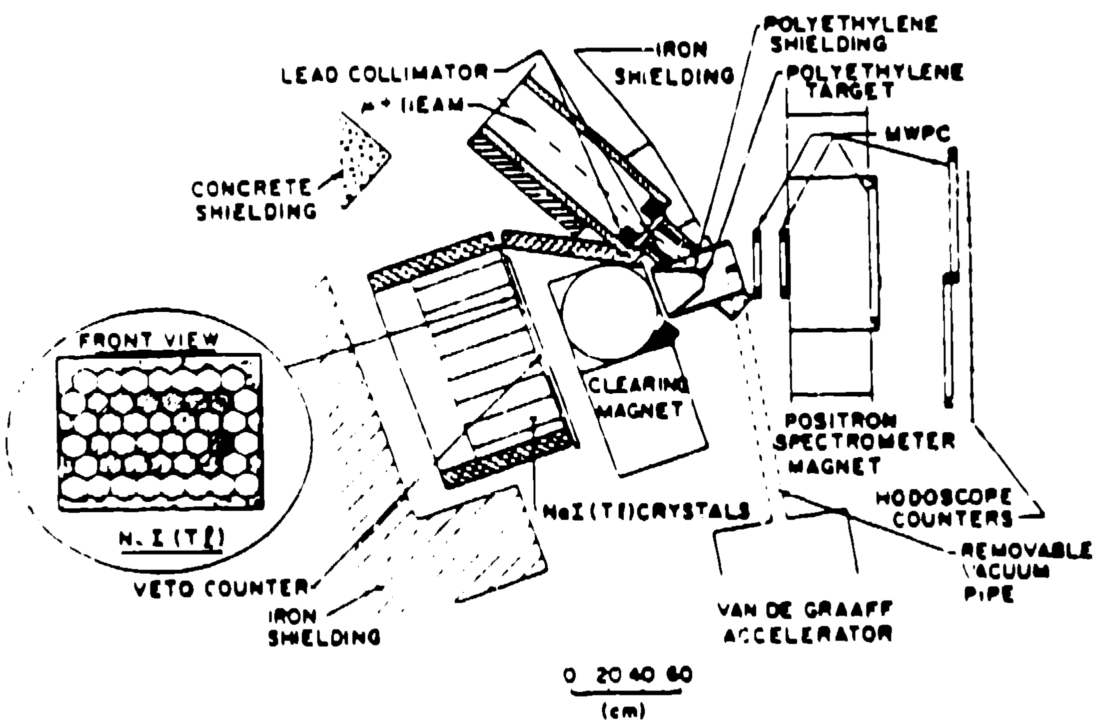


Fig. 5. The apparatus for the LAMPF $\mu \to e\gamma$ experiment.

69 mg/cm^2 with a full width at half maximum (FWHM) of only 20 mg/cm^2. The stopping intensity was 2.4×10^6 muons/s. Because of a long beam line and the low beam momentum, almost all of the pions accepted by the beam decay before reaching the experimental target. On the other hand, the channel also transports a large contamination of positrons created by pair production from γ rays in the production target. These positrons were separated from the muons by the use of a degrader in the beam, upstream of a bending magnet.

The magnetic spectrometer used MWPC's to determine the positron trajectory and a scintillation counter hodoscope to measure its time of arrival. This was not the first use of a magnetic spectrometer in a $\mu \to e\gamma$ experiment.[31]

The most novel technique used in this experiment was the photon detector. The photon telescope was a total absorption NaI detector, but it used a segmented array of NaI crystals. This permitted the measurement of the photon conversion point. In addition, a sweeping magnet was placed between the stopping target and the NaI. No positron from muon decay at the target could reach the NaI. This arrangement reduced the singles rates in the photon detector by about two orders of magnitude and allowed the experiment to use a high muon rate.

Table V shows some of the parameters of the experiment. An upper limit, $B_{\mu e\gamma} < 1.7 \times 10^{-10}$ (90% CL) was set by this experiment. The major background in this experiment was the random

TABLE V. Characteristics of Several $\mu \to e\gamma$ Experiments

	Ref. 30	Ref. 32	Ref. 33
$\left(\frac{\Omega}{4\pi}\right)$	1.8%	50%	16%
$\frac{\Delta E_\gamma}{E_\gamma}$	8% (FWHM)	6% (FWHM)	6% (FWHM)
$\frac{\Delta E_e}{E_e}$	9% (FWHM)	6% (FWHM)	0.6% (FWHM)
$\Delta t_{e\gamma}$	2ns (FWHM)	0.7 NS (FWHM)	0.6ns (FWHM)
$\Delta\theta_{e\gamma}$	5° (FWHM)	~6° (FWHM)	1.5° (FWHM)
μ^+ rate	2.5×10^6/s (average)	5 10^5/s (average)	$>10^7$/s (average)
Sensitivity	1.7×10^{-10}	10^{-11}	$<10^{-12}$

coincidence between uncorrelated positrons and photons. The internal bremsstrahlung background was at the few × 10^{-12} level.

E. Future Measurements

At LAMPF one $\mu \to e\gamma$ measurement is in progress and another is planned. The measurement in progress uses a large, multipurpose detector called the Crystal Box.[32] A schematic diagram of the apparatus is shown in Fig. 6. Surface muons stop in a thin polystyrene target in the middle of a cylindrical drift chamber. The drift chamber is surrounded by a hodoscope of plastic scintillation counters and by a large, modular, sodium iodide array. The parameters of the detector are given in Table V. The apparatus is being used to search for $\mu^+ \to e^+\gamma$, $\mu^+ \to e^+e^+e^-$, and $\mu^+ \to e^+\gamma\gamma$ simultaneously. The single particle acceptance is $\Omega/4\pi = 0.5$, which is very large. The e^+ and γ are back-to-back for $\mu^+ \to e^+\gamma$, so this is also the acceptance for this two-body decay. Since there is no magnetic field in the apparatus, the sodium iodide is exposed to the full flux of positrons from the muons decaying in the middle of the detector. In order to minimize the effects of pile-up (two particles depositing energy in an NaI crystal within the sensitive time of

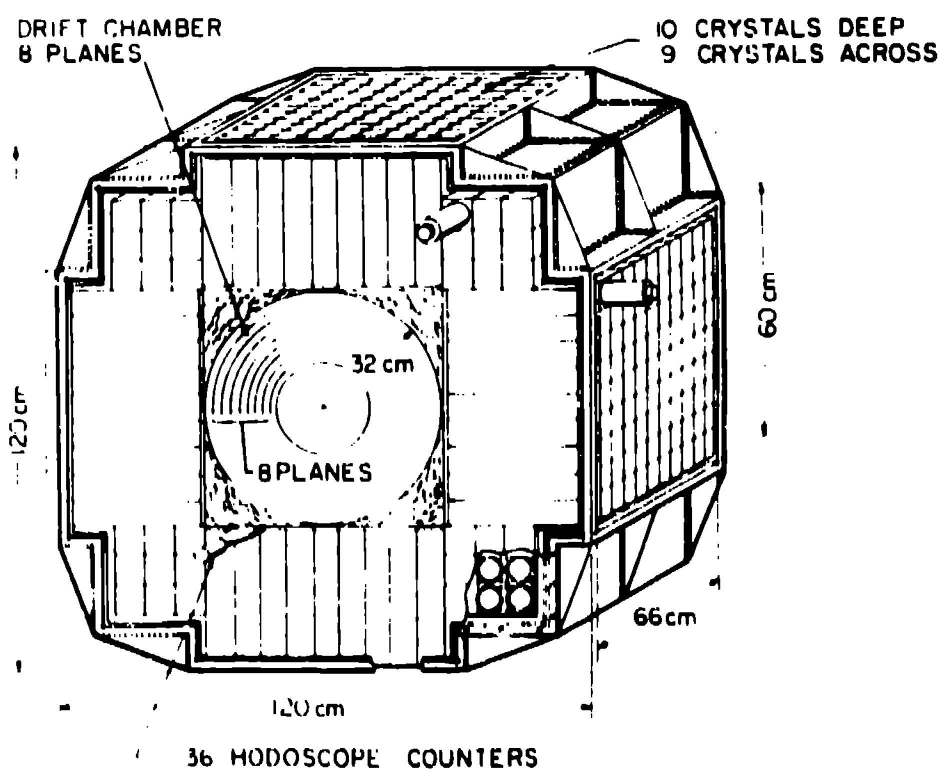


Fig. 6. Diagram of the apparatus for the Crystal Box experiment.

the energy measurement, ~200 ns), the μ^+ stopping rate is 5×10^5 μ/s, with a duty factor of ~7.5%.

The drift chamber determines the e^+ trajectory, which is traced back to the stopping target to find the muon decay point. The line from this point to the photon conversion point (determined from the energy distribution in the individual sodium iodide crystals) defines the photon trajectory. The largest background comes from the random coincidence between a positron and a photon. This should enter at a branching ratio level of a few $\times 10^{-11}$. An ultimate sensitivity of 10^{-11} for $\mu \to e\gamma$ is anticipated.

A third LAMPF $\mu \to e\gamma$ experiment is planned[33] after the Crystal Box experiment is completed, assuming a null result is obtained. The NaI will be reconfigured and combined with a magnetic spectrometer, as shown in Fig. 7. The parameters of this arrangement are given in Table '. In most respects, this is the logical extension of the first LAMPF $\mu \to e\gamma$ experiment. Better resolutions in all parameters, a higher muon flux, and a larger acceptance should result in a background-free measurement at the level of several parts in 10^{13}.

If this experiment achieves its goal but does not detect $\mu \to e\gamma$, a new design will be required to extend the sensitivity. A substantial increase in solid angle acceptance and/or muon flux

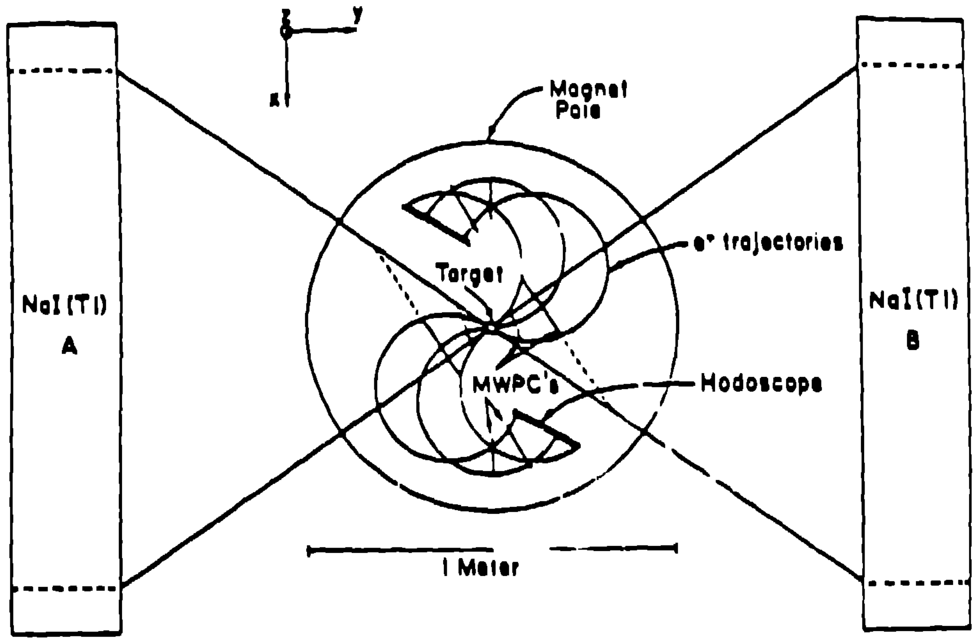


Fig. 7. Apparatus for the planned $\mu \to e\gamma$ experiment at LAMPF.

will certainly be required. As difficult as it may be to envision this at present, history indicates that it should occur. Figure 3 shows the experimental limits for various muon-number-nonconserving processes as a function of time. This figure shows the trend of an order of magnitude improvement approximately every five years.

IV. STATUS OF OTHER EXPERIMENTS

A. $\mu^+ \to e^+e^+e^-$

The present upper limit for this process comes from a 1976 experiment at Dubna.[34] This experiment utilized a multilayer cylindrical spark chamber with an axial magnetic field. The parameters of this experiment are given in Table VI. measurement are given in Table VI. The worst background came from an electron traversing the apparatus from outside (thus looking like an e^+ and an e^- emerging from the target at a relative angle of 180°) in random coincidence with an e^+ from a μ^+ decaying in the target. The constraints in this experiment are the vertex (the fact that all three charged particles must emerge from a common point in the target), timing, and conservation of energy and momentum. An intrinsic background comes from the decay $\mu^+ \to e^+e^+e^-\nu_e\bar{\nu}_\mu$. While this process satisfies the first two constraints, the expected branching ratio for events with the sum of the energies of the electron and positrons near M_μ is quite small. For example, the branching ratio for detected energy above 70 MeV is $\sim 10^{-9}$ and falls exponentially with increasing energy.[35]

Two new experiments are under way to search for $\mu \to e^+e^+e^-$. One is the aforementioned Crystal Box experiment at Los Alamos. The parameters of this measurement are given in Table VI. With no

TABLE VI. Characteristics of Several $\mu^+ \to e^+e^+e^-$ Experiments

	Ref. 34	Ref. 32	Ref. 36
$(\Delta\Omega/4\pi)$	5%	20%	16%
$\Delta E_e/E_e$	15% (FWHM)	6% (FWHM)	8% (FWHM)
$\Delta t_{ee'}$	6ns (FWHM)	0.5ns (FWHM)	0.5ns (FWHM)
Target Thickness	22 mm	1.5 mm	1.1 mm
μ^+ rate	2×10^4/s	5×10^5/s (average)	2.5×10^6/s
Sensitivity	1.9×10^{-9}	$\sim 2 \times 10^{-11}$	1.6×10^{-10} (achieved) 10^{-12} (planned)

magnetic field, there is no way to distinguish e^+ and e^-. The main background comes from three positrons from three separate muon decays in accidental coincidence. A sensitivity of $\sim 2 \times 10^{-11}$ should be reached in the next year.

The second new experiment is SINDRUM[36] at SIN. The detector, shown in Fig. 8, consists of cylindrical arrays of multiwire proportional chambers in an axial magnetic field. This design is an update of the apparatus of Korenchenko et al. (Ref. 34). The parameters of this experiment are shown in Table VI. This experiment benefits from the 100% duty factor at SIN. A preliminary run with only fou of the five chambers yielded an upper limit $B_{\mu 3e} < 1.6 \times 10^{-10}$ (90% CL). A longer run is now under way.

B. $\mu^- Z \to e^- Z$

When negative muons stop in matter, they can either decay or be captured by the nucleus, initiating the reaction $\mu^- Z \to \nu_\mu (Z - 1)$. A muon-number nonconserving process which might occur is $\mu^- Z \to e^- Z$ (called μ-e conversion). This process can be coherent (in which the nucleus remains in the ground state and so the electrons are monoenergetic with $E_e \approx m_\mu$) or incoherent (in which the nucleus is excited). Coherent μ-e conversion involves only scalar and vector quark densities.[37] Incoherent μ-e conversion involves pseudoscalar and axial vector quark densities.[38] The published limits on

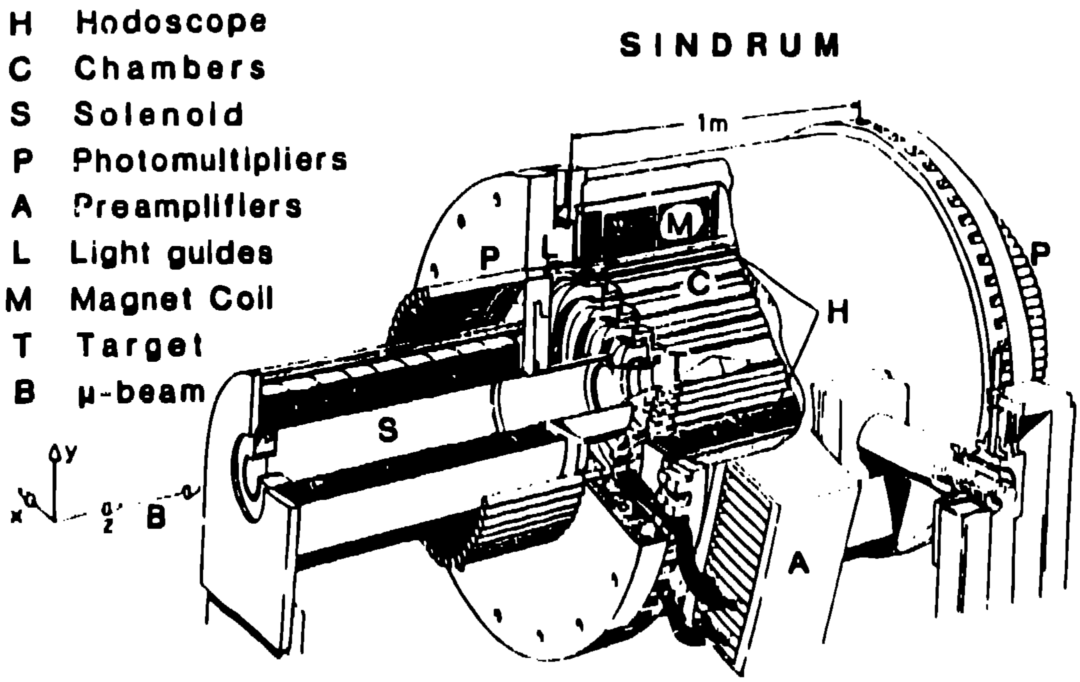


Fig. 8. Diagram for the SINDRUM experiment.

TABLE VII. Limits for Coherent $\mu^- - e^-$ Conversion

Nucleus	Limit (90% CL)	Reference
Copper	2×10^{-8}	Bryman et al., Ref. 41
Sulfur	7×10^{-11}	Badertscher et al., Ref. 39
Titanium	2×10^{-11}	Blecher et al., Ref. 40

coherent μ-e conversion for various nuclei are given in Table VII. Note that the coupling for ^{32}S must be isoscalar, whereas this is not necessarily the case for the other nuclei.

In the search for coherent μ-e conversion, one is performing a kinematically incomplete experiment to observe a rare process. The signature is a single electron with energy near 100 MeV. Backgrounds include muon decay in orbit (in which the whole atom can recoil), radiative muon capture with the production of an asymmetric $e^+ - e^-$ pair, and many possible processes induced by π^-, should any be present. The first two backgrounds fall approximately as $(E_{max} - E)^4$.

The SIN experiment[39] on ^{32}S used a streamer chamber inside an axial magnetic field to measure the electron momentum accurately. An upper limit,

$$R_{\mu e}^{coh}(^{32}S) = \frac{\Gamma(\mu^- Z \to e^- Z)_{coh}}{\Gamma(\mu^- Z \to \nu_\mu [Z - 1])} < 7 \times 10^{-11} \text{ (90\% CL)} \quad , \qquad (14)$$

was obtained. The beam in the SIN accelerator was pulsed at 400 kHz. Electrons emitted during the beam-off period were detected long after any pions had decayed or been captured.

The apparatus for a new experiment at TRIUMF[40] is shown in Fig. 9. The heart of the experiment is a time projection chamber located in a magnetic field. A preliminary limit $R_{\mu e}^{coh}(^{48}Ti) < 2 \times 10^{-11}$ (90% CL) has been obtained. The experiment is still running and hopes to achieve a sensitivity of several parts in 10^{12}. The μ-e conversion experiments at SIN, TRIUMF, and an earlier experiment at SREL[41] are compared in Table VIII.

The limits on incoherent μ-e conversion are much poorer due to the fact that the measurement is much more difficult and because no

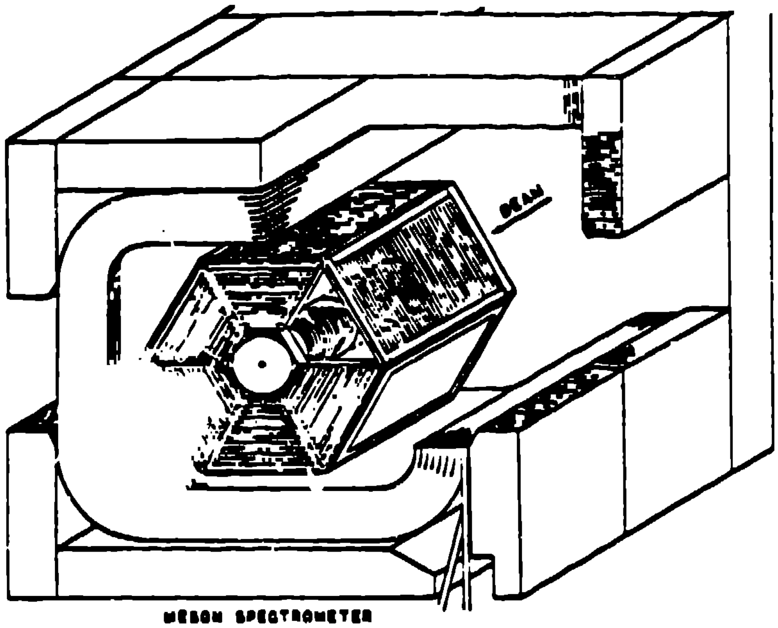


Fig. 9. Apparatus in use at TRIUMF to search for μ^- - e^- conversion.

TABLE VIII. Characteristics of Several μ^-e Conversion Experiments

	SREL (Ref. 41)	SIN (Ref. 39)	TRIUMF (Ref. 40)
$(\frac{\Omega}{4\pi})$	0.6%	5%	40%
$\frac{\Delta E_e}{E_e}$(FWHM)	~15%	7%	4%
μ^- stopping rate	~10^5	3×10^5	10^6
Sensitivity (90% CL)	1.6×10^{-8}	7×10^{-11}	2×10^{-11} (achieved) few $\times 10^{-12}$ (planned)

experiment has specifically searched for this process. Recently the SIN experiment was reanalyzed and a limit[38] $R_{\mu e}^{incoh}(^{32}S) \lesssim 8 \times 10^{-9}$ (90% CL) was obtained.

C. $\mu \to e\gamma\gamma$

The $\mu \to e\gamma\gamma$ decay can occur as bremsstrahlung from external muon and electron lines for $\mu \to e\gamma$: this would lead to an additional suppression by a factor of $\sim(\alpha/\pi)$. However, there are gauge models in which the $\mu \to e\gamma\gamma$ rate can exceed the $\mu \to e\gamma$ rate. This is the case, for example, in some theories in which the mediating heavy leptons are charged.[42] One expects a differential decay distribution given by

$$\frac{d^2\Gamma}{dE_1 dE_2} = \frac{G(a,b)}{16\pi^3 m^6} E_e E_1^2 E_2^2 (1 - \cos\theta)^2 \quad , \tag{15}$$

where $E_{1,2}$ are the photon energies, E_e is the electron energy, θ is the angle between the photons, and G describes the couplings.

In the $\mu \to e\gamma$ experiments with two large NaI detectors[28,29] $\mu \to e\gamma\gamma$ could appear because an extra photon would simply increase the measured energy without being distinguished. Thus, upper limits of $B_{\mu e\gamma\gamma} < 5 \times 10^{-8}$ (90% CL) (Ref. 42) and $B_{\mu e\gamma\gamma} < 8.4 \times 10^{-9}$ (90% CL) (Ref. 43) have been deduced.

The Crystal Box experiment[32] expects to be sensitive to $\mu \to e\gamma\gamma$ at a level of a few parts in 10^{11}. This large solid-angle detector has a much larger acceptance for $\mu \to e\gamma\gamma$: the back-to-back configuration of the existing measurements is not favored by the decay distribution of Eq. (15).

D. Some Other Rare Muon Processes

There are several other rare processes which have been studied as tests of either the Konopinski-Mahmoud or the multiplicative lepton number schemes. These processes are

$$\mu^- Z \to e^+(Z-2) \quad , \tag{16}$$

$$\mu^+ \to e^+ \bar{\nu}_e \nu_\mu \quad , \tag{17}$$

$$(\mu^+ e^-) \to (\mu^- e^+) \text{ and } e^- e^- \to \mu^- \mu^- \quad . \tag{18}$$

Present limits are (all 90% confidence levels)

$$\frac{\Gamma(\mu^- Z \to e^+[Z-2])}{\Gamma(\mu^- Z \to \nu_\mu[Z-1])} (Z = {}^{32}S) < 9 \times 10^{-10} \text{ (Ref. 39)} \quad , \tag{19}$$

and

$$\frac{\Gamma(\mu^+ \to e^+ \bar{\nu}_e \nu_\mu)}{\Gamma(\mu^+ \to e^+ \nu_e \bar{\nu}_\mu)} < 0.09 \text{ (Ref. 44)} \quad . \tag{20}$$

Limits on the processes in Eq. (18) may be found in Ref. 45.

One might think that the process $\pi^0 \to \mu e$ would also be a good place to look for muon-number nonconservation. Two recent papers[38,46] have reanalyzed old experiments and found

$$B_{\pi^0 \mu e} \equiv \frac{\Gamma(\pi^0 \to \mu^+ e^-) + \Gamma(\pi^0 \to \mu^- e^+)}{\Gamma(\pi^0 \to \text{all})} < 7 \times 10^{-8} \text{ (90\% CL)} \quad . \tag{21}$$

While this is a small number, it does not impose a meaningful constraint. The $\pi^0 \to \mu e$ decay is mediated by pseudoscalar and axial vector quark densities, as is incoherent μ-e conversion. The constraints from the incoherent conversion imply an upper limit of $\sim 10^{-13}$ for $\pi^0 \to \mu e$.[38] The reason $B_{\pi^0 \mu e}$ is so small is that the denominator in Eq. (21) is an electromagnetic rate, not a weak interaction rate.

Finally, I should mention the reaction $e^+e^- \to \mu e$.

Because one would expect an effective four-fermion inte tion, the cross section should be proportional to s. Assumi hat the muon-number violation is characterized by a coupling con t, G_X, we find[47]

$$\frac{\sigma_{ee \to \mu e}}{B_{\mu 3e}} \approx \frac{G_X^2 s}{G_X^2/G_F^2} = sG_F^2 \quad . \tag{22}$$

The present limit on $B_{\mu 3e}$ implies

$$\sigma_{ee} \lesssim sG_F^2[10^{-9}] = 4 \times 10^{-43}\ \mathrm{cm}^2 \text{ for } s = 10^4\ \mathrm{GeV}^2 \quad , \tag{23}$$

which implies an event rate of 4×10^{-11}/s at a luminosity of $10^{32}/\mathrm{cm}^2$-s.

E. Strangeness-Changing Muon-Number-Nonconserving Decays

For some theories of muon-number nonconservation, including horizontal gauge theories[48] (in which horizontal gauge interactions connect the different generations) strangeness-changing processes are the most sensitive. Present limits for some of these processes are

$$B_{K\mu e} \equiv \frac{\Gamma(K_L \to \mu e)}{\Gamma(K_L \to \mathrm{all})} < 2 \times 10^{-9} \text{ (90\% CL) (Ref. 49)} \tag{24}$$

and

$$B_{K\pi\mu e} \equiv \frac{\Gamma(K^+ \to \pi^+\mu e)}{\Gamma(K^+ \to \mathrm{all})} < 5 \times 10^{-9} \text{ (90\% CL) (Ref. 50)} \quad . \tag{25}$$

Experiments have been approved at the AGS[51] to improve these limits by several orders of magnitude. This is an area of great interest and activity. However, it lies somewhat outside the scope of this paper. The interested reader should consult Ref. 48 and references therein for more details.

F. Tau Decays

A search for lepton-number-nonconserving τ decays[52] has set upper limits at $\approx 5 \times 10^{-4}$ for processes such as $\tau \to e\gamma$, $\tau \to \mu\gamma$, $\tau \to e\mu\mu$, etc. The sensitivity of these measurements is determined by the number of available τ's. There is no obvious way to increase this sample at present. Thus, although these decays deserve as much attention as the rare muon processes, it appears that there is no way to achieve sensitivities that are as definitive as those that have been attained with muons.

V. CONCLUSIONS

The minimal standard model of the electroweak interactions has been enormously successful in accounting for a wide variety of phenomena. Nevertheless, theoretical prejudice indicates that this is not a complete theory. In the presence of any of the present possibilities beyond the minimal standard model, some of the important features of this standard model will be altered. One such feature is the conservation of lepton-family number. The cutting edge of elementary particle physics is the measurement of the parameters of the standard model and searches for extensions of the standard model. These studies can be pursued at the high-energy frontier and at the high-precision frontier. In the case of lepton-family-number conservation, the high-precision approach is the appropriate one.

Historically, the study of neutrinoless μ-e transitions has played a major role in our understanding of the fundamental interactions. The more sensitive experiments which are running or are planned will extend this understanding.

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